

Radiative Corrections to Polarized Inelastic Scattering in Coincidence

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Abstract

The complete analysis of the model-independent leading radiative corrections to cross-section and polarization observables in semi-inclusive deep-inelastic electron-nucleus scattering with detection of a proton and scattered electron in coincidence has been performed. The basis of the calculations consists of the Drell-Yan like representation in electrodynamics for both spin-independent and spin-dependent parts of the cross-section in terms of the electron structure functions. The applications to the polarization transfer effect from longitudinally polarized electron beam to detected proton as well as to scattering by the polarized target are considered.

1 Introduction

Current experiments at electron accelerators of new generation reached a new level of precision. Such a precision requires a new approach to data analysis and inclusion of all possible systematic uncertainties. One of the important sources of systematic uncertainties are electromagnetic radiative effects caused by physics processes in the next orders of perturbation theory.

The purpose of this paper is developing a unified approach to computation of radiative effects for inelastic scattering of polarized electrons in the coincidence setup, namely, when one produced hadron is detected in coincidence with the scattered electron. A broad range of measurements falls into the category of coincidence electron scattering experiments. It includes deep-inelastic semi-inclusive lepton production of hadrons, $(e, e'h)$, as well as quasielastic nucleon knock-out processes, $(e, e'N)$. The former class of experiments gives access to the flavor structure of quark-parton distributions and fragmentation functions. It is in focus of experimental programs at CERN, DESY, SLAC and JLab. Some experiments have already been completed and some are being in preparation. The detailed modern review of the activities can be found in [1]. Quasielastic nucleon knock-out allows to study single-nucleon properties in nuclear medium and probe the nuclear wave function [3, 4].

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The different theoretical aspects of strong interaction in semi-inclusive DIS were studied in a number of papers [5], [6]. The most direct experimental probe of momentum distribution in nuclei, that is presently available, is provided by means of the reaction $A(e, e'N)B$ (see reviews [7]). The particular polarization effects in a such type of reactions on the level of Born approximation with respect to the electromagnetic interaction have been investigated in Ref. [8].

There are several papers dealing with radiative effects for coincidence experiments. The lowest order correction was treated in [9] using an approach of covariant cancellation of infrared divergence. Leading log correction was studied in [10] for charm production. At last radiative correction in quasielastic scattering was recently studied in [11]. Different approaches were applied to the calculations and different approximations were done for that. These calculations adopted some specific models for structure functions. Current experimental data do not cover wide enough kinematical ranges, so extrapolation and interpolation procedures have to be used in calculating radiative effects. Therefore the model dependence of the results reduces their generality and as a result, their applicability. Furthermore higher order effects, which are important at the current level of experimental accuracies, were not systematically considered.

The method of the electron structure functions [12] allows to treat the observed cross section including both the lowest order and higher order effects, by the same way. As a result we can obtain clear and physically transparent formulae for radiative effects. In this paper we restrict our consideration to leading accuracy. It allows us to avoid an attraction of any model for the hadron structure functions and as a result to obtain some general formulae for quite wide class of the physical processes. In the case of need the NLO correction to some specific process can be obtained by standard procedure. Good examples are recent calculation of LO and NLO correction to polarization observables in DIS [13] and elastic [14] processes.

In the present paper we consider the model-independent RC to the cross-section and polarization observables in semi-inclusive deep-inelastic scattering of the longitudinally polarized electron off nucleus targets, provided that the target as well as detected hadron can be polarized. In Sec. 2 we use the electron structure function approach to calculate RC and derive the master formulae for the radiatively corrected spin-independent and spin-dependent parts of the corresponding cross-sections in the form of the Drell-Yan like representation [15] in electrodynamics. The result of this Sec. is suitable for leptonic variables when the scattered electron is detected too. In Sec. 3 we apply our master formulae to the case when polarization of the final nucleon is measured. The account of RC to the semi-inclusive DIS on the nucleus target with vector polarization has been performed in Sec. 4. In Sec. 5 we apply our approach to describe the effects of polarization transfer from the target to the detected nucleon. These effects includes both double spin (hadron-hadron) and triple spin (electron-hadron-hadron) correlations. In Sec. 6 we derive the modification of the master formulae in the case of hadronic variables (when instead of the scattered electron the total 4-momentum of the all hadrons is measured) and consider some applications. Brief discussion of the expansion of our results for the radiatively corrected polarization observables beyond the leading-log accuracy is given in Conclusion.

2 Master formula

In the recent experiment [16] the polarization transfer to the detected proton in the process with longitudinally-polarized electron beam $^{16}O(\vec{e}, e, \vec{p})^{15}N$ has been measured. This reaction is the particular case of the more general semi-inclusive deep-inelastic polarized process

$$\vec{e}^-(k_1) + A(p_1) \longrightarrow e^-(k_2) + \vec{p}(p_2) + X. \quad (1)$$

In this paper we want to clarify the question how to calculate the electromagnetic radiative corrections to the cross-section and polarization observables in the such kind of the process within the framework of the electron structure function approach.

We will use the following definition of the cross-section of the process (1) with definite spin orientation of the proton (that is detected in the final state) in terms of the leptonic and hadronic tensors

$$d\sigma = \frac{\alpha^2}{(2S_A + 1)V(2\pi)^3} \frac{L_{\mu\nu}H_{\mu\nu}}{2\hat{q}^4} \frac{d^3k_2}{\varepsilon_2} \frac{d^3p_2}{E_2}, \quad (2)$$

where S_A is the target spin, ε_2 (E_2) is the energy of the scattered electron (detected proton) and \hat{q} is the 4-momentum of the virtual photon that probes the hadron block. Hadronic tensor can be expressed via hadron electromagnetic current J_μ

$$H_{\mu\nu} = \sum_X \langle p_1 | J_\mu(\hat{q}) | p_2, X \rangle \langle X, p_2 | J_\nu(-\hat{q}) | p_1 \rangle \delta(P_x^2 - M_x^2), \quad P_x = \hat{q} + p_1 - p_2,$$

where P_x is the total 4-momentum of the undetected hadron system and M_x is its invariant mass.

The electron structure function approach yields summation of the leading-log contributions into the leptonic tensor in all orders of the perturbation theory. These contributions arise due to radiation of a hard collinear as well as the soft and virtual photons and electron-positron pairs by electrons in both, initial and final, states. In the leading approximation the electron tensor, on the right side of Eq. (2), can be written as [17]

$$L_{\mu\nu}(k_1, k_2) = \int \int \frac{dx_1 dx_2}{x_1 x_2^2} D(x_2, Q^2) [D(x_1, Q^2) \hat{Q}_{\mu\nu}^B(\hat{k}_1, \hat{k}_2) + i\lambda D_\lambda(x_1, Q^2) \hat{E}_{\mu\nu}^B(\hat{k}_1, \hat{k}_2)], \quad (3)$$

$$Q^2 = -(k_1 - k_2)^2, \quad \hat{k}_1 = x_1 k_1, \quad \hat{k}_2 = \frac{k_2}{x_2},$$

where $D(x, Q^2)$ is the structure function that describes radiation of an unpolarized electron, and $D_\lambda(x, Q^2)$ – of longitudinally-polarized one. On the level of the next-to-leading accuracy these functions differ already in the first order of the perturbation theory, but in the framework of the used here leading one, in the second order only. The corresponding difference is caused by leading contribution into D -function due to e^+e^- -pair production in the singlet channel (effect of the final-electron identity), which is different for unpolarized and longitudinally polarized electron and read [17] (KMS), [18]

$$D^s = \left(\frac{\alpha L}{2\pi}\right)^2 \left[\frac{2(1-x^3)}{3x} + \frac{1-x}{2} + (1+x) \ln x \right], \quad L = \frac{Q^2}{m_e^2},$$

$$D_\lambda^s = \left(\frac{\alpha L}{2\pi}\right)^2 \left[\frac{5(1-x)}{2} + (1+x) \ln x \right],$$

where m_e is the electron mass.

The accounting of the singlet channel contribution leads usually to very small effects (of the order 10^{-4}) because, as one can see, terms into brackets trend to compensate each other (see, for example, [19]). Below we will not distinguish between D and D_λ , which corresponds to the accounting of the nonsinglet channel contribution only (for the corresponding D -functions see [18, 19]). Such approximation allows to write compact formulae for the radiatively corrected cross-sections. We will also omit quantity Q^2 from arguments of the D -functions.

The quantity λ , on the right side of Eq. (3), is the degree of longitudinal polarization of the electron beam. The limits of the integration will be defined below. The representation (3)

follows from the quasi-real electron approximation [20]. The physical sense of variables x_1 and x_2 is as follows: $1 - x_1$ is the energy fraction of all collinear photons and e^+e^- -pairs, radiated by the initial electron, respect to its energy, $1 - x_1 = \omega/\varepsilon_1$, and quantity $(1 - x_2)/x_2$ is the same for the scattered electron.

In the Born approximation

$$Q_{\mu\nu}^B(k_1, k_2) = q^2 g_{\mu\nu} + 2(k_1 k_2)_{\mu\nu}, \quad E_{\mu\nu}^B(k_1, k_2) = 2(\mu\nu k_1 k_2), \quad (\mu\nu k_1 k_2) = \epsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}, \quad (4)$$

$$(k_1 k_2)_{\mu\nu} = k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu}, \quad q = k_1 - k_2.$$

The hadronic tensor, on the right side of Eq. (2), in general case depends on 4-momenta p_1 , p_2 , 4-momentum of the virtual photon $\hat{q} = \hat{k}_1 - \hat{k}_2$, and 4-vector of the hadron spin S that satisfies conditions: $S^2 = -1$, $(Sp_2) = 0$. For example, in the case under consideration

$$H_{\mu\nu} = H_{\mu\nu}^{(u)} + H_{\mu\nu}^{(p)},$$

$$H_{\mu\nu}^{(u)} = h_1 \tilde{g}_{\mu\nu} + h_2 \tilde{p}_{1\mu} \tilde{p}_{1\nu} + h_3 \tilde{p}_{2\mu} \tilde{p}_{2\nu} + h_4 (\tilde{p}_1 \tilde{p}_2)_{\mu\nu} + i h_5 [\tilde{p}_1 \tilde{p}_2]_{\mu\nu}, \quad (5)$$

$$H_{\mu\nu}^{(p)} = (Sp_1)[h_6 (\tilde{p}_1 N)_{\mu\nu} + i h_7 [\tilde{p}_1 N]_{\mu\nu} + h_8 (\tilde{p}_2 N)_{\mu\nu} + i h_9 [\tilde{p}_2 N]_{\mu\nu}] + (S\hat{q})[h_{10} (\tilde{p}_1 N)_{\mu\nu} + i h_{11} [\tilde{p}_1 N]_{\mu\nu} + h_{12} (\tilde{p}_2 N)_{\mu\nu} + i h_{13} [\tilde{p}_2 N]_{\mu\nu}] + (SN)[h_{14} \tilde{g}_{\mu\nu} + h_{15} \tilde{p}_{1\mu} \tilde{p}_{1\nu} + h_{16} \tilde{p}_{2\mu} \tilde{p}_{2\nu} + h_{17} (\tilde{p}_1 \tilde{p}_2)_{\mu\nu} + i h_{18} [\tilde{p}_1 \tilde{p}_2]_{\mu\nu}], \quad N_\mu = \epsilon_{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} \hat{q}_\sigma = (\mu p_1 p_2 \hat{q}), \quad [ab]_{\mu\nu} = a_\mu b_\nu - a_\nu b_\mu, \quad (6)$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{\hat{q}_\mu \hat{q}_\nu}{\hat{q}^2}, \quad \tilde{p}_{i\mu} = p_{i\mu} - \frac{(\hat{q} p_i) \hat{q}_\mu}{\hat{q}^2}, \quad i = 1, 2,$$

where h_i ($i = 1 - 18$) are the hadron semi-inclusive structure functions which depend in general on four invariants. These invariants can be taken as \hat{q}^2 , $(\hat{q} p_1)$, $(\hat{q} p_2)$ and $(p_1 p_2)$.

The j-component of the proton polarization P^j , that could be measured in experiment, is defined as the ratio of the spin-dependent part of the cross-section (2) (which is caused by contraction of the leptonic tensor with the spin-dependent part of the hadronic one $H_{\mu\nu}^{(p)}$, with the given j-component of the proton spin) to the spin-independent one (which is caused by contraction of $L_{\mu\nu}$ with $H_{\mu\nu}^{(u)}$)

$$P^j = \frac{d\sigma^{(p)}(\lambda, S^j, k_1, k_2, p_1, p_2)}{d\sigma^{(u)}(\lambda, k_1, k_2, p_1, p_2)}. \quad (7)$$

Note that P^j is non-zero even if $\lambda = 0$ (the case of unpolarized electron beam) due to non-zero single-spin correlations in semi-inclusive processes.

In the process (1) one can measure, in principle, three independent components: P^l (longitudinal), P^t (transverse) and P^n (normal), which could be taken respect to definite physical directions and planes created by 3-momenta of the particles participating in the process. If any additional particle (photons and e^+e^- -pairs), radiated by electrons with 4-momenta k_1 and k_2 , is not detected, there are three independent directions: along \vec{p}_2 , \vec{k}_1 and \vec{k}_2 . In this case any components of the proton polarization as well as the corresponding proton spin components S^j will be defined for the Born kinematics and their directions are not affected by radiation.

Combining formulae for the cross-section (2), the definitions of the lepton (3,4) and hadron (5,6) tensors and taking into account the last discussions, we can write the following representation for the cross-section of the process (1)

$$\varepsilon_2 E_2 \frac{d\sigma(\lambda, S^j, k_1, k_2, p_1, p_2)}{d^3 k_2 d^3 p_2} = \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \frac{d\sigma^B(\lambda, S^j, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3 \hat{k}_2 d^3 p_2}, \quad (8)$$

where $j = l, t, n$. The factor $1/x_1$ that enters into definition of $L_{\mu\nu}$ is absorbed into flow in the reduced Born cross-section that equals by definition (see Eq. (2))

$$\hat{\varepsilon}_2 E_2 \frac{d\sigma^B(\lambda, S^j, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2} = \frac{\alpha^2}{(2S_A + 1)\hat{V}(2\pi)^3} \frac{L_{\mu\nu}^B(\hat{k}_1, \hat{k}_2, \lambda) H_{\mu\nu}(S^j, \hat{q}, p_1, p_2)}{2\hat{q}^4},$$

where $\hat{V} = x_1 V$. With the chosen accuracy the representation (8) is valid for both spin-dependent ($d\sigma^{(p)}$) and spin-independent ($d\sigma^{(u)}$) parts of the cross-section.

In theoretical calculations it is useful often to parameterize the proton spin 4-vector, which enters in definition of the hadron tensor, in terms of the particle 4-momenta [21]. In considered case we have four 4-momenta to express any component of the proton spin S^j in a such way that

$$S^j = S^j(k_1, k_2, p_1, p_2). \quad (9)$$

Let us imagine for a moment that chosen parameterization on the right side of Eq. (9) is stabilized relative substitution

$$k_1 \rightarrow \hat{k}_1, \quad k_2 \rightarrow \hat{k}_2, \quad S^{js}(k_1, k_2, p_1, p_2) = S^{js}(\hat{k}_1, \hat{k}_2, p_1, p_2).$$

(Further we will label such stabilized parameterizations by the index with small letter). In this case we can write the Born cross-section under integral sign on the right side of Eq. (8) in the form

$$\hat{\varepsilon}_2 E_2 \frac{d\sigma^B(\lambda, S^j, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2} = \hat{\varepsilon}_2 E_2 \frac{d\sigma_j^B(\lambda, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2}. \quad (10)$$

If the proton spin S^j is unstable under above substitution (in this case we will use the index with capital letter) it can be expressed always in terms of stabilized one by means of orthogonal matrix

$$S^J(k_1, k_2, p_1, p_2) = A_{Jj}(k_1, k_2, p_1, p_2) S^j(\hat{k}_1, \hat{k}_2, p_1, p_2), \quad A_{Jj} = -S^J S^j. \quad (11)$$

Using the last formula and taking into account that in the considered class of the processes the hadron tensor depends linearly on the proton spin, we can write the master representation for the spin-dependent part ($d\sigma^{(p)}$) of the cross-section of the process (1) for arbitrary orientation of the proton spin in the following form

$$\varepsilon_2 E_2 \frac{d\sigma(\lambda, S^J, k_1, k_2, p_1, p_2)}{d^3k_2 d^3p_2} = A_{Jj} \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \frac{d\sigma_j^B(\lambda, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2}, \quad (12)$$

where we bear in mind the summation over index $j = l, t, n$.

This representation is the electrodynamical analogue of the well known in QCD Drell-Yan formula [15], that was applied earlier to calculate the electromagnetic radiative corrections to the total cross-section of the electron-positron annihilation into hadrons [18], to small-angle Bhabha scattering cross-section at LEP1 [19], to unpolarized [22] and polarized deep-inelastic cross-sections [13], and to polarized elastic electron-proton scattering [14]. In the next Section we will show how this representation can be used to describe the leading radiative corrections in polarized semi-inclusive deep-inelastic events. It is obvious that in the framework of the leading accuracy one needs to find the adequate parameterizations of the proton spin 4-vector, to calculate the elements of the orthogonal matrix A_{Jj} , derive the spin-independent and spin-dependent parts of the Born cross-section for given parameterization S^j , and determine the limits of the integration over x_1 and x_2 in the master formula (12).

3 Analysis of semi-inclusive deep-inelastic events with polarization transfer

Let us begin with the parameterizations of the proton spin 4-vector in process (1). To describe this process we will use the following set of invariant variables

$$z = \frac{2p_1 p_2}{V}, \quad z_{1,2} = \frac{2k_{1,2} p_2}{V}, \quad y = \frac{2p_1(k_1 - k_2)}{V}, \quad x = \frac{-q^2}{2p_1 q}, \quad V = 2p_1 k_1, \quad q = k_1 - k_2. \quad (13)$$

It is physically justified to determine the longitudinal component of the proton spin along direction of $-\vec{p}_1$ as seen from the rest frame of the detected proton. This direction does not affected by the lepton collinear radiation and the corresponding parameterization has a form

$$S_\mu^l = \frac{z p_{2\mu} - 2\tau_2 p_{1\mu}}{m \sqrt{z^2 - 4\tau_1 \tau_2}}, \quad \tau_1 = \frac{M^2}{V}, \quad \tau_2 = \frac{m^2}{V}, \quad (14)$$

where $M(m)$ is the mass of the target nucleus (detected proton). It is easy to verify that in the rest frame of proton ($p_2 = (m, 0)$) this longitudinal component equals to $(0, -\vec{n}_1)$, where $\vec{n}_1 = \vec{p}_1/|\vec{p}_1|$, and in the lab. system ($p_1 = (M, 0)$) it equals to $(|\vec{p}_2|, E_2 \vec{n}_2)/m$, where \vec{n}_2 is the unit vector in direction of the detected proton 3-momentum.

For the fixed longitudinal component we have a few possibilities to determine the transverse and normal ones. First, take the transverse component in the plane (\vec{k}_1, \vec{p}_2) and the normal component in the plane that is perpendicular to it. Orientations of these planes do not change during substitution $\vec{k}_1 \rightarrow \hat{\vec{k}}_1$, therefore in this case we have

$$S_\mu^t = \frac{(z^2 - 4\tau_1 \tau_2) k_{1\mu} + (2z_1 \tau_1 - z) p_{2\mu} + (2\tau_2 - z z_1) p_{1\mu}}{\sqrt{V(z^2 - 4\tau_1 \tau_2)[1]}}, \quad S_\mu^n = \frac{2(\mu k_1 p_1 p_2)}{\sqrt{V^3[1]}}, \quad (15)$$

$$[1] = z z_1 - \tau_2 - z_1^2 \tau_1, \quad (S^j S^i) = -\delta_{ji}.$$

By full analogy with above procedure we can determine other stabilized set of transverse and normal components relative to the plane (\vec{k}_2, \vec{p}_2)

$$\tilde{S}_\mu^t = \frac{(z^2 - 4\tau_1 \tau_2) k_{2\mu} + (2z_2 \tau_1 - z(1 - y)) p_{2\mu} + (2\tau_2(1 - y) - z z_2) p_{1\mu}}{\sqrt{V(z^2 - 4\tau_1 \tau_2)[2]}}, \quad (16)$$

$$\tilde{S}_\mu^n = \frac{2(\mu k_2 p_1 p_2)}{\sqrt{V^3[2]}}, \quad [2] = z z_2(1 - y) - \tau_2(1 - y)^2 - z_2^2 \tau_1.$$

The sets (15) and (16) represent the complete list of the stabilized parameterizations of the proton spin components on the condition that the longitudinal component is chosen according to Eq. (14). There are a lot of unstable parameterizations because we can take them relative to arbitrary plane $(a\vec{k}_1 + b\vec{k}_2, \vec{p}_2)$ with arbitrary numbers a and b . In further we will consider the physically favorable set with $a = -b = 1$ only. The corresponding transverse and normal components read

$$S_\mu^T = \frac{(z^2 - 4\tau_1 \tau_2) q_\mu + (2(z_1 - z_2) \tau_1 - z y) p_{2\mu} + (2y \tau_2 - z(z_1 - z_2)) p_{1\mu}}{\sqrt{V(z^2 - 4\tau_1 \tau_2)[q]}}, \quad (17)$$

$$S_\mu^N = \frac{2(\mu q p_1 p_2)}{\sqrt{V^3[q]}} , \quad [q] = zy(z_1 - z_2) + xy(z^2 - 4\tau_1\tau_2) - (z_1 - z_2)^2\tau_1 - y^2\tau_2 .$$

Let us consider now the relation between stabilized (for definiteness we will work with the set (15)) set and unstable one. It is obvious that this relation can be written as follows

$$S^N = \cos\theta S^n + \sin\theta S^t , \quad S^T = -\sin\theta S^n + \cos\theta S^t , \quad (18)$$

where

$$\cos\theta = -(S^N S^n) = -(S^T S^t) = \frac{z(z_1(1+y) - z_2) + xy(z^2 - 4\tau_1\tau_2) - 2z_1(z_1 - z_2)\tau_1 - 2y\tau_2}{2\sqrt{[1][q]}} ,$$

$$\sin\theta = -(S^N S^t) = (S^T S^n) = \frac{\eta}{2} \sqrt{\frac{(z^2 - 4\tau_1\tau_2)}{[1][q]}} ,$$

$$\eta = \text{sign}[(p_1 p_2 k_1 k_2)] \sqrt{\frac{16}{V^4} (p_1 p_2 k_1 k_2)^2} , \quad (p_1 p_2 k_1 k_2) = \epsilon_{\mu\nu\rho\sigma} p_{1\mu} p_{2\nu} k_{1\rho} k_{2\sigma} ,$$

$$\frac{16(p_1 p_2 k_1 k_2)^2}{V^4} = x^2 y^2 (4\tau_1\tau_2 - z^2) + 2xy[z(z_2 + z_1(1-y)) - 2z_1 z_2 \tau_1 - 2(1-y)\tau_2] - (z_2 - z_1(1-y))^2 .$$

One can verify that the necessary condition $\cos^2\theta + \sin^2\theta = 1$ is satisfied.

Now we can write down the spin-independent (we bear in mind that it means independent on the proton spin only) and spin-dependent parts of the cross-section of the process (1) as

$$\varepsilon_2 E_2 \frac{d\sigma_{(u),L}}{d^3 k_2 d^3 p_2} = \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \frac{d\hat{\sigma}_{(u),l}^B}{d^3 \hat{k}_2 d^3 p_2} , \quad (19)$$

$$\varepsilon_2 E_2 \frac{d\sigma_N}{d^3 k_2 d^3 p_2} = \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \left[\cos\theta \frac{d\hat{\sigma}_n^B}{d^3 \hat{k}_2 d^3 p_2} + \sin\theta \frac{d\hat{\sigma}_t^B}{d^3 \hat{k}_2 d^3 p_2} \right] , \quad (20)$$

$$\varepsilon_2 E_2 \frac{d\sigma_T}{d^3 k_2 d^3 p_2} = \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \left[-\sin\theta \frac{d\hat{\sigma}_n^B}{d^3 \hat{k}_2 d^3 p_2} + \cos\theta \frac{d\hat{\sigma}_t^B}{d^3 \hat{k}_2 d^3 p_2} \right] , \quad (21)$$

where $d\hat{\sigma}^B$, with any low index, denotes the corresponding Born cross-section given at *shifted* values of $k_{1,2} \rightarrow \hat{k}_{1,2}$. The corresponding *shifted* dimensionless variables, introduced by relation (13), read

$$\hat{x} = \frac{x_1 xy}{x_1 x_2 + y - 1} , \quad \hat{y} = \frac{x_1 x_2 + y - 1}{x_1 x_2} , \quad \hat{V} = x_1 V , \quad \hat{z} = \frac{z}{x_1} , \quad \hat{z}_1 = z_1 , \quad \hat{z}_2 = \frac{z_2}{x_1 x_2} . \quad (22)$$

Eqs.(19)–(21) are the straightforward consequences of the master representation (12). In order to obtain $d\sigma_n$ and $d\sigma_t$ on the left side of Eqs. (20) and (21) we have to take obviously $\cos\theta = 1$, $\sin\theta = 0$.

Now we must derive the Born cross-sections which enter on the right sides of Eqs. (19)–(21). The spin-independent part of the cross-section for longitudinally- polarized electron beam (with degree λ) is expressed in terms of the hadron structure functions $h_1 - h_5$ as

$$\varepsilon_2 E_2 \frac{d\sigma_{(u)}^B}{d^3 k_2 d^3 p_2} = \frac{\alpha^2 V}{2(2S_A + 1)(2\pi)^3 q^4} H_1 , \quad (23)$$

$$H_1 = -\frac{2xy}{V}h_1 + (1 - y - xy\tau_1)h_2 + (z_1z_2 - xy\tau_2)h_3 + (z_2 + z_1(1 - y) - xyz)h_4 - \lambda\eta h_5 .$$

Note that the phase space of the detected proton also can be expressed in terms of invariant variables (13)

$$\frac{d^3p_2}{E_2} = \frac{V}{2|\eta|}dz_1dz_2dz . \quad (24)$$

If the proton spin is directed along S^l then the spin-dependent part of the Born cross-section reads

$$\varepsilon_2 E_2 \frac{d\sigma_l^B}{d^3k_2 d^3p_2} = -\frac{\alpha^2 V^3 \eta \sqrt{z^2 - 4\tau_1\tau_2}}{8(2S_A + 1)m(2\pi)^3 q^4} \left[H_2 + \frac{[z(z_1 - z_2) - 2y\tau_2]}{z^2 - 4\tau_1\tau_2} H_3 \right] , \quad (25)$$

$$H_2 = (2 - y)h_6 + (z_1 + z_2)h_8 + \frac{\lambda}{\eta}(\eta_1 h_7 + \eta_2 h_9) ,$$

$$H_3 = (2 - y)h_{10} + (z_1 + z_2)h_{12} + \frac{\lambda}{\eta}(\eta_1 h_{11} + \eta_2 h_{13}) ,$$

$$\eta_1 = y[z_2 - z_1(1 - y) - xz(2 - y) + 2x(z_1 + z_2)\tau_1] ,$$

$$\eta_2 = (z_1 - z_2)(z_2 - z_1(1 - y)) + xyz(z_1 + z_2) - 2xy(2 - y)\tau_2 .$$

In the case of transverse orientation of the proton spin (along S^t) we have

$$\varepsilon_2 E_2 \frac{d\sigma_t^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2 \eta}{8(2S_A + 1)(2\pi)^3 q^4} \sqrt{\frac{V}{z^2 - 4\tau_1\tau_2}} \left[\psi H_3 - \frac{z^2 - 4\tau_1\tau_2}{\sqrt{[1]}} H_4 \right] , \quad (26)$$

$$\psi = \frac{xy(z^2 - 4\tau_1\tau_2) + (z - 2z_1\tau_1)(z_1 - z_2) + (zz_1 - 2\tau_2)y}{\sqrt{[1]}} = 2\sqrt{[q]} \cos \theta ,$$

where H_4 can be obtained from H_1 by means of simple replacement $h_i \rightarrow h_{i+13}$.

At last, for the normal orientation of the proton spin (along S^n) the spin-dependent part of the cross-section of the process (1) reads

$$\varepsilon_2 E_2 \frac{d\sigma_n^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2 \sqrt{V}}{8(2S_A + 1)(2\pi)^3 q^4} \left[-\frac{\eta^2}{\sqrt{[1]}} H_3 - \psi H_4 \right] . \quad (27)$$

We have to determine also the limits of integration over variables x_1 and x_2 in the master representation (12). They can be obtained from the condition that the semi-inclusive deep-inelastic process takes place. For an electron-proton scattering it is possible on the condition that the hadron state consists, at least, of a proton and a pion. This leads to inequality

$$x_1 x_2 + y - 1 - x_1 x y \geq x_2 \delta , \quad \delta = \frac{(m + m_\pi)^2 - m^2}{V} , \quad (28)$$

where m_π is the pion mass. This inequality yields for the limits

$$1 > x_2 > \frac{1 - y + xyx_1}{x_1 - \delta} , \quad 1 > x_1 > \frac{1 + \delta - y}{1 - xy} . \quad (29)$$

For the electron-nucleus scattering process (1) that is considered here we must change the pion mass, in definition of δ , by the bound energy of the ejected proton in a given nucleus.

It is interesting to note that in the case, when the polarizations of the final proton are measured relative to stabilized orientations, the corresponding Born values and the leading

radiative corrections to them are expressed in terms of the same hadron structure functions. The situation changes radically if one measures polarizations relative to the unstable orientations. In this case the contributions to the polarizations, caused by the radiative corrections due to hard collinear radiation, are expressed in terms of another sets of hadron structure functions as compared with the Born polarizations. To give this fact more transparent, we write down the spin-dependent part of the Born cross-section for the orientations of the proton spin along S^N and S^T

$$\varepsilon_2 E_2 \frac{d\sigma_T^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2 \eta}{4(2S_A + 1)(2\pi)^3 q^4} \sqrt{\frac{V[q]}{z^2 - 4\tau_1 \tau_2}} H_3 , \quad (30)$$

$$\varepsilon_2 E_2 \frac{d\sigma_N^B}{d^3k_2 d^3p_2} = -\frac{\alpha^2 V^2 \sqrt{V[q]}}{4(2S_A + 1)(2\pi)^3 q^4} H_4 . \quad (31)$$

These formulae can be derived from Eqs. (20) and (21) if to take $D(x_i)$ -functions in form of δ -function, which corresponds to the radiationless process (or to the Born approximation).

4 Semi-inclusive deep-inelastic scattering on polarized target

In this section we will apply the master representation to the analysis of polarized phenomena in semi-inclusive deep-inelastic scattering of polarized nucleus

$$\vec{e}^+(k_1) + \vec{A}(p_1) \rightarrow e^-(k_2) + H(p_2) + X , \quad (32)$$

where H is arbitrary hadron and nucleus A has definite vector polarization P . In this case the leptonic tensor is as before (see Eqs. (3) and (4)), and the hadronic tensor has the same structure as defined by Eqs. (5) and (6), where one needs to use polarization of the nucleus P instead of the proton spin S and write (Pp_2) instead of (Sp_1) . Besides, we will use the notation $g_1 - g_{18}$ for the corresponding hadron structure functions.

Usually when studying the polarization phenomena the various asymmetries are measured and to find them it is necessary to know the polarization-independent and polarization-dependent parts of the cross-section at different orientations of the target polarization. Therefore, the corresponding analysis can be performed in the same manner as it was done in Section 2.

Let us, at first, define the parameterizations of the nucleus polarization 4-vector in terms of 4-momenta. As a stabilized set we can choose longitudinal and transverse components as given in Ref. [13]

$$P_\mu^l = \frac{2\tau_1 k_{1\mu} - p_{1\mu}}{M} , \quad P_\mu^t = \frac{k_{2\mu} - (1 - y - 2xy\tau_1)k_{1\mu} - xyp_{1\mu}}{\sqrt{Vxy(1 - y - xy\tau_1)}} , \quad (33)$$

and for normal component we use

$$P_\mu^n = \frac{2(\mu k_1 k_2 p_1)}{\sqrt{V^3 xy(1 - y - xy\tau_1)}} . \quad (34)$$

It is easy to verify that parameterizations (33), (34) are not changed at the substitution $k_{1,2} \rightarrow \hat{k}_{1,2}$. In lab. system this set corresponds to direction of the longitudinal polarization along \vec{k}_1 ,

the transverse polarization is in the plane (\vec{k}_1, \vec{k}_2) and the normal one is in the plane, that is perpendicular to (\vec{k}_1, \vec{k}_2) plane.

Another set of the polarizations can be chosen in a such way that longitudinal component will be along \vec{q} -direction in lab. system and the transverse one is in the plane (\vec{q}, \vec{k}_1) . In this case the normal component coincides with (34) and

$$P_\mu^L = \frac{2\tau_1(k_{1\mu} - k_{2\mu}) - yp_{1\mu}}{M\sqrt{y^2 + 4xy\tau_1}}, \quad P_\mu^T = \frac{(1 + 2x\tau_1)k_{2\mu} - (1 - y - 2x\tau_1)k_{1\mu} - x(2 - y)p_{1\mu}}{\sqrt{Vx(1 - y - xy\tau_1)(y + 4x\tau_1)}}. \quad (35)$$

The sets (35) and (33) are transformed one to other by orthogonal matrix

$$P^L = \cos\theta_1 P^l + \sin\theta_1 P^t, \quad P^T = -\sin\theta_1 P^l + \cos\theta_1 P^t, \\ \cos\theta_1 = \frac{y(1 + 2x\tau_1)}{\sqrt{y(y + 4x\tau_1)}}, \quad \sin\theta_1 = -2\sqrt{\frac{x\tau_1(1 - y - xy\tau_1)}{y + 4x\tau_1}}. \quad (36)$$

The master equation (12) can be applied to the polarization-independent part of the cross-section (32) as well as to the polarization-dependent one. Therefore, we have to derive the Born cross-section for the stabilized set. The simple calculation gives

$$\varepsilon_2 E_2 \frac{d\sigma_{(u)}^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V}{(2S_A + 1)(2\pi)^3 q^4} G_1. \quad (37)$$

Note that numerical coefficient in front of G_1 is twice as much as compared with that on the right side of Eq. (23) in front of H_1 . The reason is that in this case we do not fix the spin state of the final hadron H .

The polarization-dependent part of the cross-section for the longitudinal stabilized polarization reads

$$\varepsilon_2 E_2 \frac{d\sigma_l^B}{d^3k_2 d^3p_2} = -\frac{\alpha^2 V^3 \eta}{4(2S_A + 1)M(2\pi)^3 q^4} [(2\tau_1 z_1 - z)G_2 - y(1 + 2x\tau_1)G_3 + 2\tau_1 G_4], \quad (38)$$

where the functions G_i , $i = 1 - 4$, can be derived from H_i by replacement the hadron structure functions g_j instead of h_j .

The corresponding part of the cross-section in the case of the transverse polarization can be written as follows

$$\varepsilon_2 E_2 \frac{d\sigma_t^B}{d^3k_2 d^3p_2} = -\frac{\alpha^2 V^2 \eta \sqrt{Vxy(1 - y - xy\tau_1)}}{4(2S_A + 1)(2\pi)^3 q^4} \left[\frac{z_2 - xyz - z_1(1 - y - 2xy\tau_1)}{xy(1 - y - xy\tau_1)} G_2 + \right. \\ \left. 2G_3 + \frac{1 + 2x\tau_1}{x(1 - y - xy\tau_1)} G_4 \right]. \quad (39)$$

For the normal polarization the spin-dependent part of the cross-section is

$$\varepsilon_2 E_2 \frac{d\sigma_n^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^2}{4(2S_A + 1)(2\pi)^3 q^4} \sqrt{\frac{V}{xy(1 - y - xy\tau_1)}} [\eta^2 G_2 - y(z_2(1 + 2x\tau_1) - \\ z_1(1 - y - 2x\tau_1) - xz(2 - y))G_4]. \quad (40)$$

The application of the master representation (12) leads to following expressions for radiatively corrected (with the leading accuracy) contributions to the cross-section of the process (32)

$$\varepsilon_2 E_2 \frac{d\sigma_{(u),N}}{d^3k_2 d^3p_2} = \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \frac{d\hat{\sigma}_{(u),n}^B}{d^3\hat{k}_2 d^3p_2}, \quad (41)$$

$$\varepsilon_2 E_2 \frac{d\sigma_L}{d^3k_2 d^3p_2} = \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \left[\cos \theta_1 \frac{d\hat{\sigma}_l^B}{d^3\hat{k}_2 d^3p_2} + \sin \theta_1 \frac{d\hat{\sigma}_t^B}{d^3\hat{k}_2 d^3p_2} \right], \quad (42)$$

$$\varepsilon_2 E_2 \frac{d\sigma_T}{d^3k_2 d^3p_2} = \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \left[-\sin \theta_1 \frac{d\hat{\sigma}_l^B}{d^3\hat{k}_2 d^3p_2} + \cos \theta_1 \frac{d\hat{\sigma}_t^B}{d^3\hat{k}_2 d^3p_2} \right]. \quad (43)$$

Let us write also the cross-sections, on the left sides of Eqs.(42) and (43), in the Born approximation

$$\varepsilon_2 E_2 \frac{d\sigma_L^B}{d^3k_2 d^3p_2} = \frac{\alpha^2 V^3 \eta}{4(2S_A + 1)(2\pi)^3 M q^4} \left[\frac{yz - 2(z_1 - z_2)\tau_1}{\sqrt{y(y + 4x\tau_1)}} G_2 + \sqrt{y(y + 4x\tau_1)} G_3 \right], \quad (44)$$

$$\begin{aligned} \varepsilon_2 E_2 \frac{d\sigma_T^B}{d^3k_2 d^3p_2} = & \frac{\alpha^2 V^2 \eta \sqrt{V}}{4(2S_A + 1)(2\pi)^3 q^4} \left[-\sqrt{\frac{y + 4x\tau_1}{x(1 - y - xy\tau_1)}} G_4 + \right. \\ & \left. \frac{xz(2 - y) - z_2 + z_1(1 - y) - 2x\tau_1(z_1 + z_2)}{\sqrt{x(y + 4x\tau_1)(1 - y - xy\tau_1)}} G_2 \right]. \end{aligned} \quad (45)$$

As one can see, the polarization-dependent parts of the Born cross-section consist of less number of the hadron structure functions as compared with radiatively corrected ones.

We can also use the 4-vector p_2 to parameterize the nucleus polarization 4-vector. If to choose the longitudinal polarization along \vec{p}_2 in the lab. system, then the stabilized set may be defined with respect to the plane (\vec{k}_1, \vec{p}_2) and unstable one with respect to the plane (\vec{q}, \vec{p}_2) as in Section 2, and the corresponding calculations are very close to given there. But parameterizations, used in this Section, look more physically and they can be used also to describe the polarization phenomena in inclusive deep-inelastic events.

5 Polarization transfer from target to detected proton

Let us consider effects of the polarization transfer from the vector polarized target to detected proton in the process

$$\vec{e}^-(k_1) + \vec{A}(p_1) \rightarrow e^-(k_2) + \vec{p}(p_2) + X \quad (46)$$

for the case of longitudinally polarized electron beam and vector polarization of the target.

The general form of the hadronic tensor in this case reads

$$H_{\mu\nu} = H_{\mu\nu}^{(u)} + H_{\mu\nu}^{(S)} + H_{\mu\nu}^{(W)} + H_{\mu\nu}^{(SW)}, \quad (47)$$

where $S(W)$ labels the vector polarization of the target (spin of the detected proton). All the effects caused by the first three terms on the right side of Eq. (47) were considered in previous Sections and now we will investigate the radiative corrections to the hadron double-spin correlations which arise just due to the last term

$$H_{\mu\nu}^{(SW)} = (Sp_2)(Wp_1)[f_1 \tilde{g}_{\mu\nu} + f_2 \tilde{p}_{1\mu} \tilde{p}_{1\nu} + f_3 \tilde{p}_{2\mu} \tilde{p}_{2\nu} + f_4 (\tilde{p}_1 \tilde{p}_2)_{\mu\nu} + i f_5 [\tilde{p}_1 \tilde{p}_2]_{\mu\nu}] +$$

$$\begin{aligned}
& (Sp_2)(Wq)[f_6\tilde{g}_{\mu\nu} + f_7\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_8\tilde{p}_{2\mu}\tilde{p}_{2\nu} + f_9(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_{10}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] + \\
& (Sp_2)(WN)[f_{11}(\tilde{p}_1N)_{\mu\nu} + if_{12}[\tilde{p}_1N]_{\mu\nu} + f_{13}(\tilde{p}_2N)_{\mu\nu} + if_{14}[\tilde{p}_2N]_{\mu\nu}] + \\
& (Sq)(Wp_1)[f_{15}\tilde{g}_{\mu\nu} + f_{16}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_{17}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + f_{18}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_{19}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] + \\
& (Sq)(Wq)[f_{20}\tilde{g}_{\mu\nu} + f_{21}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_{22}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + f_{23}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_{24}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] + \\
& (Sq)(WN)[f_{25}(\tilde{p}_1N)_{\mu\nu} + if_{26}[\tilde{p}_1N]_{\mu\nu} + f_{27}(\tilde{p}_2N)_{\mu\nu} + if_{28}[\tilde{p}_2N]_{\mu\nu}] + \\
& (SN)(Wp_1)[f_{29}(\tilde{p}_1N)_{\mu\nu} + if_{30}[\tilde{p}_1N]_{\mu\nu} + f_{31}(\tilde{p}_2N)_{\mu\nu} + if_{32}[\tilde{p}_2N]_{\mu\nu}] + \\
& (SN)(Wq)[f_{33}(\tilde{p}_1N)_{\mu\nu} + if_{34}[\tilde{p}_1N]_{\mu\nu} + f_{35}(\tilde{p}_2N)_{\mu\nu} + if_{36}[\tilde{p}_2N]_{\mu\nu}] + \\
& (SN)(WN)[f_{37}\tilde{g}_{\mu\nu} + f_{38}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + f_{39}\tilde{p}_{2\mu}\tilde{p}_{2\nu} + f_{40}(\tilde{p}_1\tilde{p}_2)_{\mu\nu} + if_{41}[\tilde{p}_1\tilde{p}_2]_{\mu\nu}] .
\end{aligned} \tag{48}$$

Thus, the coefficients of the polarization transfer from the target to the detected proton are described, in general, by 41 structure functions. If the electron beam is unpolarized, then the symmetrical part of the hadronic tensor contributes only, and this corresponds to double-spin (hadron-hadron) correlations in the cross-section of the process (46). The antisymmetric part of the hadron tensor contributes in the case of longitudinally-polarized electron beam due to triple-spin (electron-hadron-hadron) correlations.

The corresponding radiatively corrected parts of the cross-section for the unstable orientations of the target nucleus polarization S^J (given by Eq. (35)) and detected proton spin W^I (given by Eq. (17)) can be written as follows

$$\varepsilon_2 E_2 \frac{d\sigma_{JI}}{d^3k_2 d^3p_2} = \sum_{j,i} A_{Jj} B_{Ii} \int \int \frac{dx_1 dx_2}{x_2^2} D(x_1) D(x_2) \hat{\varepsilon}_2 E_2 \frac{d\hat{\sigma}_{ji}^B}{d^3\hat{k}_2 d^3p_2} \tag{49}$$

where the Born cross-section under integral sign is defined for the stable orientations of S^j (given by Eqs. (33), (34)) and W^i (given by Eqs. (14), (15)) and depends on the *shifted* variables

$$\hat{\varepsilon}_2 E_2 \frac{d\hat{\sigma}_{ji}^B}{d^3\hat{k}_2 d^3p_2} = \hat{\varepsilon}_2 E_2 \frac{d\sigma^B(\lambda, S^j, W^i, \hat{k}_1, \hat{k}_2, p_1, p_2)}{d^3\hat{k}_2 d^3p_2} .$$

In accordance with the calculations in Sections 3 and 4, matrices A_{Jj} and B_{Ii} are

$$A_{Jj} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \quad B_{Ii} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{50}$$

$$I, J = L, T, N, \quad i, j = l, t, n .$$

If we will write the hadron-hadron spin correlations in the Born cross-section as

$$\varepsilon_2 E_2 \frac{d\sigma_{ji}^B}{d^2k_2 d^3p_2} = \frac{\alpha^2 V^4 X_{ji}}{16(2\pi)^3 2(2S_A + 1)q^4}, \tag{51}$$

then the quantities X_{ji} can be written in the form

$$X_{ll} = 2\sqrt{\frac{f\tau_1}{\tau_2}} \left\{ \eta^2 (R_{29} + \xi R_{33}) + \frac{2}{V^2 \tau_1} [b(F_1 + \xi F_6) - d(F_{15} + \xi F_{20})] \right\}, \tag{52}$$

$$X_{lt} = \eta^2 \sqrt{\frac{f}{\tau_1 [1]}} [bR_{11} - dR_{25} + 2\tau_1 F_{37} - \frac{2\psi}{\eta^2 V^2 f} \sqrt{[1]} (2bF_6 - 2dF_{20} + \eta^2 V^2 \tau_1 R_{33})], \tag{53}$$

$$X_{ln} = \frac{\eta}{\sqrt{\tau_1}} [\psi(bR_{11} - dR_{25} + 2\tau_1 F_{37}) + \frac{2}{V^2 \sqrt{[1]}} (2bF_6 - 2dF_{20} + \eta^2 V^2 \tau_1 R_{33})] , \quad (54)$$

$$X_{ul} = \sqrt{\frac{f}{r\tau_2}} \{ \eta^2 d(R_{29} + \xi R_{33}) + \frac{4}{V^2} [2r(F_{15} + \xi F_{20}) + \zeta(F_1 + \xi F_6)] \} , \quad (55)$$

$$X_{tt} = \eta^2 \sqrt{\frac{f}{r[1]}} [\zeta R_{11} + 2rR_{25} + dF_{37} - \frac{\psi \sqrt{[1]}}{\eta^2 V^2 f} (\eta^2 V^2 dR_{33} + 4\zeta F_6 + 8rF_{20})] , \quad (56)$$

$$X_{tn} = \frac{\eta}{\sqrt{r}} [\psi(\zeta R_{11} + 2rR_{25} + dF_{37}) + \frac{1}{V^2 \sqrt{[1]}} (\eta^2 V^2 dR_{33} + 4\zeta F_6 + 8rF_{20})] , \quad (57)$$

$$X_{nl} = \eta \sqrt{\frac{f}{r\tau_2}} [\eta_1(R_{29} + \xi R_{33}) - \frac{4}{V^2} (F_1 + \xi F_6)] , \quad (58)$$

$$X_{nt} = \frac{\eta}{\sqrt{fr}} [\psi(\frac{4}{V^2} F_6 - \eta_1 R_{33}) + \frac{f}{\sqrt{[1]}} (\eta_1 F_{37} - \eta^2 R_{11})] , \quad (59)$$

$$X_{nn} = -\frac{\eta^2}{\sqrt{r}} [\frac{1}{\sqrt{[1]}} (\frac{4}{V^2} F_6 - \eta_1 R_{33}) - \psi(\frac{\eta_1}{\eta^2} F_{37} - R_{11})] . \quad (60)$$

Here we used the following short notation

$$b = 2z_1\tau_1 - z, \quad d = y(1 + 2x\tau_1), \quad f = z^2 - 4\tau_1\tau_2, \quad r = xy(1 - y - xy\tau_1),$$

$$\zeta = z_2 - z_1(1 - y - 2xy\tau_1) - xyz, \quad \xi = \frac{z(z_1 - z_2) - 2y\tau_2}{z^2 - 4\tau_1\tau_2} .$$

Functions R_l and F_l , which enter in the expressions for X_{ji} , are defined by means of the hadron structure function f 's in Eq. (48) as

$$R_l = (2 - y)f_l + (z_1 + z_2)f_{l+2} + \frac{\lambda}{\eta}(\eta_1 f_{l+1} + \eta_2 f_{l+3}) , \quad (61)$$

$$F_l = -\frac{2xy}{V}f_l + (1 - y - xy\tau_1)f_{l+1} + (z_1 z_2 - xy\tau_2)f_{l+2} + (z_2 + z_1(1 - y) - xyz)f_{l+3} - \lambda\eta f_{l+4} . \quad (62)$$

6 Hadronic variables

There exist the experimental possibility to measure the total 4-momentum of the hadron system X instead to record the scattered electron in semi-inclusive reactions. In such experiments the momentum q_h of heavy intermediate photon, that probes the hadron structure, can be determined explicitly. The corresponding set of dynamical variables is labeled usually as hadronic one.

In the case of the hadronic variables we have to eliminate the phase space of the scattered electron and introduce the heavy photon phase space by using the identity

$$\frac{d^3 k_2}{\varepsilon_2} = 2x_2^2 x_h \frac{d^4 q_h}{Q_h^2} \delta(x_1 - x_h), \quad \frac{d^4 q_h}{Q_h^2} = \frac{dQ_h^2 dx_h dy_h dz_h}{4x_h^2 |\eta_h|} , \quad (63)$$

$$x_h = -\frac{Q_h^2}{2k_1 q_h} , \quad y_h = \frac{2p_1 q_h}{V} , \quad z_h = \frac{2p_2 q_h}{V} , \quad Q_h^2 = -q_h^2 ,$$

$$\eta_h^2 = \frac{Q_h^2}{V} \left[(4\tau_1\tau_2 - z^2) \frac{Q_h^2}{x_h^2 V} + 2(1 - \frac{y_h}{x_h})(zz_1 - 2\tau_2) + 2(z_1 - \frac{z_h}{x_h})(z - 2z_1\tau_1) \right] - (z_h - z_1y_h)^2 .$$

Therefore, by combining representation (3) for the leptonic tensor and (63), as well as bearing in mind the independence of the hadronic tensor on variable x_2 , the expression for the quantity $L_{\mu\nu}d^3k_2/\varepsilon_2$, in the case of the hadronic variables can be written as follows

$$\frac{d^3k_2}{\varepsilon_2} L_{\mu\nu} = \frac{D(x_h, Q_h^2)}{x_h^2} L_{\mu\nu}^B(\hat{k}_1, \hat{k}_1 - q_h, \lambda) \frac{dx_h dy_h dz_h dQ_h^2}{2|\eta_h|} . \quad (64)$$

Note that for the events with undetected scattered electron the lower limit of the integration over x_2 in Eq. (3) equals to 0. In accordance with the Kinoshita–Lee–Nauenberg theorem [23], the mass singularities caused by the final–state radiation would disappear in this case. On the language of the electron structure functions this fact exhibits itself due to relation

$$\int_0^1 D(x, Q^2) dx = 1 ,$$

which was used to write Eq. (64).

The lepton tensor in the Born approximation can be rewritten as

$$L_{\mu\nu}^B(k_1, k_1 - q_h) = 2(k_1 q_h) \tilde{g}_{\mu\nu} + 4\tilde{k}_{1\mu} \tilde{k}_{1\nu} - 2i\lambda(\mu\nu k_1 q_h) , \quad (65)$$

and the physically–founded parameterizations for S^j in the process (1) and P^j in the process (32) remain now stable with respect to the scale transformation $k_1 \rightarrow x_h k_1$. For example, one set can be chosen as given by Eqs. (14), (15) and other as

$$S_{h\mu}^L = S_{\mu}^l, \quad S_{h\mu}^T = \frac{(z^2 - 4\tau_1\tau_2)q_{h\mu} + (2z_h\tau_1 - zy_h)p_{2\mu} + (2y_h\tau_2 - zz_h)p_{1\mu}}{\sqrt{V(z^2 - 4\tau_1\tau_2)[q_h]}}, \quad S_{h\mu}^N = \frac{2(\mu q_h p_1 p_2)}{\sqrt{V^3[q_h]}} ,$$

$$[q_h] = zz_h y_h + \frac{Q_h^2}{V}(z^2 - 4\tau_1\tau_2) - z_h^2\tau_1 - y_h^2\tau_2, \quad (66)$$

with the transverse component in the plane (\vec{q}_h, \vec{p}_2) in lab. system.

Two physical sets of the target polarizations, both with the normal component perpendicular to the plane (\vec{k}_1, \vec{q}_h) , may be chosen as

$$P_{h\mu}^l = \frac{2\tau_1 k_{1\mu} - p_{1\mu}}{M}, \quad P_{h\mu}^t = \left[-q_{h\mu} + (y_h + \frac{2Q_h^2\tau_1}{x_h V})k_{1\mu} - \frac{Q_h^2}{x_h V}p_{1\mu} \right] K^{-1}, \quad P_{h\mu}^n = \frac{-2(\mu k_1 q_h p_1)}{VK} , \quad (67)$$

with the longitudinal component along \vec{k}_1 in lab. system and

$$P_{h\mu}^L = \frac{2\tau_1 q_{h\mu} - y_h p_{1\mu}}{MG}, \quad P_{h\mu}^T = \left[(y_h^2 + 4\tau_1 \frac{Q_h^2}{V})k_{1\mu} - (y_h + \frac{2Q_h^2\tau_1}{x_h V})q_{h\mu} - \frac{Q_h^2}{V}(2 - \frac{y_h}{x_h})p_{1\mu} \right] (KG)^{-1} ,$$

$$P_{h\mu}^N = P_{h\mu}^n, \quad K = \sqrt{Q_h^2(1 - \frac{y_h}{x_h} - \frac{Q_h^2\tau_1}{x_h^2 V})}, \quad G = \sqrt{y_h^2 + 4\frac{Q_h^2\tau_1}{V}} , \quad (68)$$

with the longitudinal component along \vec{q}_h . The different components of the P_h^j in lab. system are

$$P_h^L = (0, \vec{n}_q), \quad P_h^T = (0, \frac{\vec{n}_1 - (\vec{n}_1 \vec{n}_q) \vec{n}_q}{\sqrt{1 - (\vec{n}_1 \vec{n}_q)^2}}), \quad P_h^N = (0, \frac{[\vec{n}_q \times \vec{n}_1]}{\sqrt{1 - (\vec{n}_1 \vec{n}_q)^2}}),$$

$$\vec{n}_q = \frac{\vec{q}_h}{|\vec{q}_h|}, \quad \vec{n}_1 = \frac{\vec{k}_1}{|\vec{k}_1|}.$$

All these sets of proton spin and target polarization given by Eqs. (66), (67) and Eq. (68), are stable with respect to the initial-state collinear radiation. This can be verified by replacement $x_h k_1$ instead of k_1 at which

$$k_1 \rightarrow x_h k_1, \quad x_h \rightarrow 1, \quad y_h \rightarrow \frac{y_h}{x_h}, \quad z_h \rightarrow \frac{z_h}{x_h}, \quad z \rightarrow \frac{z}{x_h}, \quad V \rightarrow x_h V, \quad \tau_{1,2} \rightarrow \frac{\tau_{1,2}}{x_h}. \quad (69)$$

To make the invariance of P^j ($j = l, t, n$) and P^J ($J = L, T, N$) under replacement (69) more transparent one can express x_h in terms of Q_h^2 and $(k_1 q_h)$. Then, for example,

$$K = \sqrt{Q_h^2 + y_h 2(k_1 q_h) - \frac{4(k_1 q_h)^2 \tau_1}{V}},$$

and it is easy to see that this quantity is not changed under the substitution (69). Note also that quantity η_h can be derived by means of the rule

$$\eta_h = x_h \eta^*,$$

where η^* is determined from η with substitution Q_h^2/V instead of xy , $z_1 - z_h$ instead of z_2 and subsequent replacement (69).

That is why the cross-section for both the spin-independent and spin-dependent parts in the case of the hadronic variables can be written in the following form

$$E_2 \frac{d\sigma^j}{d^3 p_2 dQ_h^2 dx_h dy_h dz_h} = \frac{D(x_h, Q_h^2)}{x_h^2} E_2 \frac{d\hat{\sigma}_j^B}{d^3 p_2 dQ_h^2 d\hat{y}_h d\hat{z}_h}, \quad (70)$$

where

$$E_2 \frac{d\hat{\sigma}_j^B}{d^3 p_2 dQ_h^2 d\hat{y}_h d\hat{z}_h} = \frac{\alpha^2 C}{(2\pi)^3 (2S_A + 1) \hat{V} Q_h^4 2|\eta^*|} L_{\mu\nu}(\hat{k}_1, \hat{k}_1 - q_h, \lambda) H_{\mu\nu}(q_h, p_1, p_2; S^j(P^j)).$$

Here C equals 1/2 (or 1) for process (1) or (32).

The representation (70) shows that the using of the hadron variables allows to tag the initial-state radiated photon. Indeed, for fixed 4-momentum P_x one can reconstruct 4-momentum q_h and, consequently, the variable x_h which is the energy fraction of the photon radiated by the initial electron (see Eq. (63)).

The Born cross-section on the right side of Eq. (70) has the form that is very like to the corresponding cross-section for the leptonic variables. We can formulate the following rules to write it:

i) change phase space differentials in the left sides of the expressions valid for the leptonic variables

$$\frac{\varepsilon_2}{d^3 k_2} \rightarrow \frac{2|\eta_{1h}|}{dQ_h^2 dy_h dz_h}, \quad \eta_{1h} = \eta_h(x_h = 1),$$

ii) apply substitution

$$xy \rightarrow \frac{Q_h^2}{V}, \quad y \rightarrow y_h, \quad z_2 \rightarrow z_1 - z_h$$

to the right sides.

These rules lead, for example, to the formula for the spin-dependent part of the cross-section of the process (1) in the case of the longitudinal polarization (which follows from Eq. (25))

$$E_2 \frac{d\sigma_L^B}{d^3p_2 dQ_h^2 dy_h dz_h} = -\frac{\alpha^2 V^3 \eta_{1h} \sqrt{z^2 - 4\tau_1 \tau_2}}{8m(2S_A + 1)(2\pi)^3 Q_h^4 2|\eta_{1h}|} \left[H_2^{(h)} + \frac{zz_h - 2y_h \tau_2}{z^2 - 4\tau_1 \tau_2} H_3^{(h)} \right], \quad (71)$$

$$H_2^{(h)} = (2 - y_h)h_6 + (2z_1 - z_h)h_8 + \frac{\lambda}{\eta_{1h}}(\eta_1^{(h)} h_7 + \eta_2^{(h)} h_9),$$

$$\eta_1^{(h)} = \frac{Q_h^2}{V} [2(2z_1 - z_h)\tau_1 - z(2 - y_h)] + z_1 y_h^2 - z_h y_h, \quad \eta_2^{(h)} = \frac{Q_h^2}{V} [z(2z_1 - z_h) - 2(2 - y_h)\tau_2] - z_h^2 + z_1 z_h y_h,$$

where $H_3^{(h)}$ is derived from $H_2^{(h)}$ by the change $h_i \rightarrow h_{i+4}$.

The spin-dependent part of the cross-section of the process (32) for the case of the normal target polarization (that follows from Eq. (40) reads

$$E_2 \frac{d\sigma_N^B}{d^3p_2 dQ_h^2 dy_h dz_h} = -\frac{\alpha^2 V^3}{4(2S_A + 1)(2\pi)^3 Q_h^4 K(x_h = 1) 2|\eta_{1h}|} \left\{ \eta_{1h}^2 G_2^{(h)} - [y_h(z_1 y_h - z_h) + \right. \quad (72)$$

$$\left. \frac{Q_h^2}{V} (2\tau_1(2z_1 - z_h) - z(2 - y_h))] G_4^{(h)} \right\}.$$

The rest of the spin-dependent and spin-independent parts of the cross-sections for processes (1) and (32) can be obtained by full analogy using the above rules and results given in Sections 3, 4.

The variable x_h characterizes the inelasticity of the initial-state electron, and in the absence of radiation it equals to 1. The electron structure function $D(x_h, Q_h^2)$ has singularity at $x_h = 1$, and representation (70) shows that this singularity is such that

$$\lim_{x_h \rightarrow 1} D(x_h, Q_h^2) dx_h = 1 \quad (73)$$

because in this limiting case the left side of Eq. (70), being multiplied by dx_h , have to coincide with the Born cross-section.

7 Conclusion

In this paper we consider RC to the polarization observables in a wide class of semi-inclusive deep-inelastic processes. We restrict ourselves to the leading-log accuracy and neglect the contribution of the pair production in the singlet channel. This gives the possibility to write the compact formulae for the radiatively corrected spin-independent and spin-dependent parts of the corresponding cross-sections in the form of the Drell-Yan representation in electrodynamics by means of the electron structure functions. The parameterization of the hadron spin 4-vectors in terms of the particles 4-momenta is very important during the calculations. If the momentum of the intermediate photon that probes the hadron structure, is determined in terms of the hadronic variables, the traces of the final-state radiation disappear in the final result in the framework of used approximation.

In practice the corrections can be computed adopting some specific model for structure functions. In this case the correction gets some model dependence that can contribute to the systematical error in experimental measurements. Another way is related to some iteration

procedure, when the fit of processed experimental data is used for this required model. We note that obtained leading log formulae have a partly factorized form, being quite convenient for this procedure. The examples for DIS case can be found in [21, 24].

Apart from the discussed classes of experiments the results can be also adopted to exclusive electroproduction processes, when the unobservable hadron state is one particle. In this case structure functions include an additional δ -function, so some analytical manipulations could be necessary.

Sometimes the accuracy more than the leading one is necessary. To go beyond the leading accuracy one must modify the master representations. This modification concerns both the electron structure function and cross-section (hard part) that depends on the shifted variables. To improve the hard part, it is enough to take into account the radiation of single additional non-collinear photon and to add the non-leading part of the one-loop correction. The corresponding procedure is described in Ref. [22] for unpolarized deep-inelastic scattering and in Ref. [21] (AAM) for quasi-elastic polarized electron-proton scattering. To be complete one needs also to improve the structure functions by the addition of the second order next-to-leading contributions caused by double collinear photon emission and pair production. Besides, the non-leading contributions into D -function caused by the one-loop corrected collinear single-photon emission and two-loop correction have to be added properly. These contributions are different for symmetric and asymmetric parts of the leptonic tensor and can be extracted from the results given in Ref. [17] (for two-loop correction, see [25]). So, in this case we have to distinguish between D and D_λ yet at the level of the nonsinglet channel contribution. The concrete calculations will be done elsewhere.

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